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DCV-003-1161003

Seat No. _______

M. Sc. (Sem. I) Examination

August - 2022

Mathematics : CMT - 1003

(Topology - I)

Faculty Code : 003

Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions from the following.

(2) There are total ten questions.

(3) Each question carries equal (14) marks.

1 Answer the following :

(1) Define with example : Topology on a set.

(2) Define with example :

(a) Discrete Topology.

(b) Indiscrete Topology.

(3) Is the collection $\mathfrak{B} = \{\{x\} / x \in X\}$ is basis for any set X or not? Justify your answer.

(4) Define with example : Usual Topology on $\mathbb{R}$.

(5) Define with example : Order Topology.

(6) Justify whether every sequence in an indiscrete topological space is convergent or not?

(7) Define with example : Continuous Function.

2 Answer the following :

(1) Let X be any topological space and Y be indiscrete topological space. Is every function $f : X \to Y$ continuous? Justify your answer.

(2) Define with example : Euclidean Metric.

(3) Let X and Y be topological spaces. Consider the function $\pi_2 : X \to Y$ defined by $\pi_2(x, y) = y$, for all $(x, y) \in X \times Y$. Is π_2 is continuous? Justify your answer.

(4) Define with example : Interior point of a set.

(5) Define with example : Metric on a set.

(6) Define with example : Linear Continuum.

(7) Define with example : Uniform Convergence.

3 Answer the followings :

(a) On the set of real numbers $\mathbb{R}$, define $\tau_f = \{U \subseteq \mathbb{R} / \mathbb{R} - U$ is either finite or all of $\mathbb{R}\}$. Prove that, τ_f is topology on $\mathbb{R}$.

(b) State and prove, Pasting Lemma.

4 Answer the followings :

(a) Let X be a topological space. Suppose $\mathcal{C}$ is a collection of open sets such that for each open set U and $x \in U$, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then prove that, $\mathcal{C}$ is basis for the topology of X.

(b) Let X be a topological space and $\mathfrak{B}$ be a basis for the topology on X. Let Y be a subspace of X. Define $\mathfrak{B}_Y = \{B \cap Y \mid B \in \mathfrak{B}\}$. Prove that, $\mathfrak{B}_Y$ is basis for the subspace topology.

5 Answer the followings :

(a) Let X be any non-empty set and let $\{\tau_\beta\}_{\beta \in I}$ be a collection of topologies on X. Prove that, $\bigcap_{\beta \in I} \tau_\beta$ is topology on X. Also prove that is $S \subset P(X)$, then there is smallest topology on X which contains S.

(b) Let X and Y be topological spaces and π_1, π_2 be the projection maps. Prove that,

$$\mathcal{S} = \left\{\pi_1^{-1}(U) / U \text{ is open in } X\right\} \cup \left\{\pi_2^{-1}(V) : V \text{ is open in } Y\right\}$$

is a subbasis for the product topology on $X \times Y$.

6 Answer the followings :

(a) Let X be a topological space and Y be a subspace of X. Let A be a subset of Y and $\bar{A}$ denote the closure of A in X. Prove that, the closure of A in Y equals $\bar{A} \cap Y$.

(b) Prove that, the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology of $\mathbb{R}$, but are not comparable with each other.

7 Answer the followings :

(a) Let X and Y be topological spaces. Let $\mathcal{S}$ be a sub basis for the topology on Y. Let $f: X \to Y$ be a function. Prove that, f is continuous if and only if $f^{-1}(S)$ is open in X, for every $S \in \mathcal{S}$.

(b) State and prove, Uniform limit theorem.

8 Answer the followings :

(a) Let X and Y be topological spaces. Let $f: X \to Y$ be a function. If for every closed set B in Y, the set $f^{-1}(B)$ is closed in X then prove that, for each $x \in X$ and each neighbourhood V of x such that $f(U) \subseteq V$.

(b) Let X and Y be topological spaces. Let $f: X \to Y$ be a function. If X can be written as the union of open sets U_α such that $f|_{U_\alpha}$ is continuous for each α then prove that, f is continuous.

9 Answer the followings :

(a) State and prove, Sequence Lemma.

(b) Let (X, d) be metric space. Prove that, $\mathcal{B}_d = \{B_d(x, r) / x \in X, r > 0\}$ is a basis for the metric space (X, d).

10 Answer the followings :

(a) Prove that, the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

(b) Let X and Y be topological spaces. Prove that, if $f : X \to Y$ is a continuous function then for every sequence $\{x_n\}$ converging to x, the sequence $\{f(x_n)\}$ converges to $f(x)$. Also prove that the converse holds if X is metrizable.
